Medical Image Segmentation Based on Variational Bayes

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Outline

• Motivation
• Brief introduction of Variational Bayes
• Variational Inference for Mixture model
  • Mixture of Gaussian model
  • Finite Student’s t-mixture
  • Infinite Student’s t-mixture
  • Experiment
• Laplacian Regularized Gaussian mixture model
  • Laplacian Regularization
  • Experiment
• Summary
Motivation

• Application
  • Medical Image analysis
  • 3D reconstruction
  • Data compression
  • Image understanding

• Approach
  • Region segmentation
  • Edge-Detection
  • Markov random Field
  • Clustering-Based (mixture model)
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Variational Bayes

Key Notes:
- Distributional Approximation
- By minimizing the KL divergence.
- Mean Field assumption
- Bayesian frameworks

\[
Q(Z_i) \propto \frac{1}{C} \exp \left\{ \ln P(Z_i, Z_{-i}, D) \right\}_{Q(Z_{-i}) \text{or} Q(mb(Z_i))}
\]
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mixture of Gaussian

\[ p(X \mid Z, \mu, \Lambda) = \prod_{n=1}^{N} \prod_{k=1}^{K} N(x_n \mid \mu_k, \Lambda_k^{-1})^{z_{nk}} \]

\[ p(X, Z, \pi, \mu, \Lambda) = p(X \mid Z, \mu, \Lambda) p(Z \mid \pi) p(\pi) p(\mu \mid \Lambda) p(\Lambda) \]
Student’s t-mixture model

\[ p(x_n \mid s, \{\mu, \Lambda, \nu\}) = \sum_{m=1}^{M} St(x_n \mid \mu_m, \Lambda_m, \nu_m)^{s_m} \]
Infinite Student’s t-mixture

\[ DP(\alpha, G_0) \]

\[ G = \sum_{j=1}^{\infty} \pi_j(V) \delta_{\Theta_j} \quad \pi_j(V) = V_j \prod_{i=1}^{j-1} (1 - V_i) \]

\[ V_j \sim \text{Beta}(1, \alpha) \]

\[ p(\alpha) = \text{Gam}(\alpha \mid \eta_1, \eta_2) \]

\[ p(X) = \prod_{n=1}^{N} \sum_{j=1}^{\infty} \pi_j(V) \cdot \text{St}(x_n \mid \mu_j, \Lambda_j, v_j) \]
Example: Variational Inference for GMM

\[ q^*(\pi) \sim \text{Dir}(\alpha) \quad \alpha = \alpha_0 + N_k \quad N_k = \sum_{n=1}^{N} r_{nk} \]

\[ q^*(\mu_k, \Lambda_k) = N(\mu_k | m_k, (\beta_k \Lambda_k)^{-1})W(\Lambda_k | w_k, \nu_k) \]

\[ \beta_k = \beta_0 + N_k, m_k = \frac{1}{\beta_k} (\beta_0 m_0 + N_k \bar{x}_k), \nu_k = \nu_0 + N_k, \bar{x}_k = \frac{1}{N_k} \sum_{n=1}^{N} r_{nk} x_n \]

\[ w_k^{-1} = w_0^{-1} + N_k S_k + \frac{\beta_0 N_k}{\beta_0 + N_k} (\bar{x}_k - m_0)(\bar{x}_k - m_0)^T, S_k = \frac{1}{N_k} \sum_{n=1}^{N} r_{nk} (x_k - x_n)(x_k - x_n)^T \]

\[ q^*(Z) = \prod_{n=1}^{N} \prod_{k=1}^{K} r_{nk} z_{nk} \]

\[ r_{nk} \propto \tilde{\pi}_k \tilde{\Lambda}_k^{1/2} \exp \left\{ -\frac{D}{2 \beta_k} - \frac{\nu_k}{2} (x_n - m_k)^T W_k (x_n - m_k) \right\} \]
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Experiment

• Data: Internet Brain Segmentation Repository (IBSR)¹, including 20 low resolution T1-weighted brain MRI images.

• Task: Segment MRI into Gray Matter (GM), White Matter (WM) and Cerebrospinal Fluid (CSF)

• Measure: Jaccard similarity coefficient (JSC)

• MATLAB toolbox (preprocessing): SPM8

¹ Center for Morphometric Analysis at Massachusetts General Hospital, “The Internet Brain Segmentation Repository (IBSR),” http://www.cma.mgh.harvard.edu/ibsr/index.html, Jan. 2009
Segmentation Result

Source Data

(a) groundTruth

(b) EM-GMM

(c) VB-GMM

(d) VB-SMM

(e) VB-iSMM
Algorithm complexity:

\[ O(tKNd^3) \]

Iteration Times:

\[ t_{VB} = (1/10 \sim 1/2)t_{EM} \]
Segmentation Accuracy

(a) JSC of CSF

(b) JSC of GM

accuracy is not reduced.

(c) JSC of WM
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Laplacian regularization

- Manifold assumption.
- Construction
  - $p$-nearest neighbors graph
  - Assign weight matrix $S$
  - Laplacian graph: $L = D - S$

\[
\min R_k = \frac{1}{2} \sum_{i,j=1}^{m} (P(k \mid x_i) - P(k \mid x_j))^2 S_{ij}
\]

\[
\max L(\Theta) = \log P(X \mid \Theta) - \lambda \sum_{k=1}^{K} R_k
\]
A Toy Example: Two Moons Pattern

(a) Original data
(b) K-means
(c) VB-GMM
(d) VB-lapGMM
Segmentation Result

Source Data

(a) groundTruth

(b) EM-GMM

(c) VB-GMM

(d) VB-SMM

(e) VB-iSMM

(f) EM-lapGMM

(g) VB-lapGMM

(h) VB-lapSMM

(i) VB-lapiSMM
Accuracy: vbgmm vs. vblapgmm

(a) Cerebrospinal Fluid

(b) Gray matter

(c) White matter

(d) Total

Improve Accuracy
Result: SMM/iSMM vs. GMM

(a) top-left: VB-GMM vs. VB-lapGMM
(b) top-right: VB-SMM vs. VB-lapSMM
(c) lower-left: VB-iSMM vs. VB-lapiSMM

Robust with outlier
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Variational Bayes vs. Expectation Maximization
- Reduce the iteration times, 1/10~1/2

finite/Infinite Students’ t-mixture model
- Reduce noise, more robust

Variational laplacian regularized mixture model
- Improve accuracy, enhance stability
Thank you